

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/22

Paper 2, maximum raw mark 80

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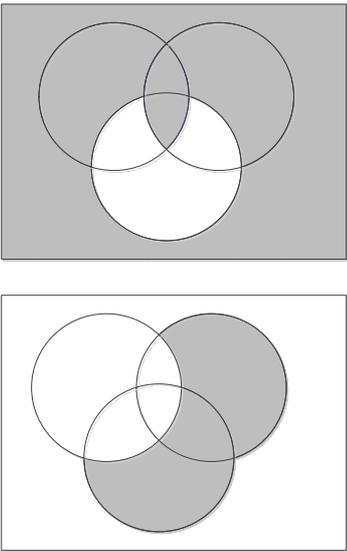
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<p>1 (a)</p>  <p>(b)</p>	<p>No. in H only = $50 - x$; No in F only = $60 - x$ Sum: $50 - x + 60 - x + x + 30 - 2x = 98$</p> <p style="text-align: center;">$x = 14$</p>	<p>B1</p> <p>B1</p> <p>B1 M1 A1</p>	<p>Both written or on diagram Add at least 3 terms each with x involved and equate to 98 so</p>
<p>2</p>	$9x^2 + 2x - 1 < (x + 1)^2$ $8x^2 < 2 \quad \text{oe isw}$ $-\frac{1}{2} < x < \frac{1}{2}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>Expand and collect terms</p>
<p>3</p>	$\log_2(x + 3) = \log_2 y + 2 \rightarrow x + 3 = 4y$ $\log_2(x + y) = 3 \rightarrow x + y = 8$ $x + 3 = 4(8 - x)$ $5x = 29 \rightarrow x = 5.8, \quad \text{oe}$ $y = 2.2 \quad \text{oe}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Eliminate y or x from two linear three term equations</p>

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4	(i)	$f(37) = 3$ or $gf(x) = \frac{\sqrt{x-1}-3-2}{2(\sqrt{x-1}-3)-3}$ $gf(37) = \frac{3-2}{6-3} = \frac{1}{3}$	B1	
	(ii)	$y = \sqrt{x-1} - 3 \rightarrow (y+3)^2 = x-1$ $(x+3)^2 + 1 = f^{-1}(x)$ oe isw	M1 A1	Rearrange and square in any order Interchange x and y and complete
	(iii)	$y = \frac{x-2}{2x-3}$ $2xy - 3y = x - 2 \rightarrow 2xy - x = 3y - 2$ $\frac{3x-2}{2x-1} = g^{-1}(x)$ oe	M1 A1	Multiply and collect like terms Interchange and complete Mark final answer
5	(i)	$B = 900$	B1	
	(ii)	$B = 500 + 400e^2 = 3455$ or 3456 or 3460	B1	3455.6 scores B0
	(iii)	$\left(\frac{dB}{dt}\right) = 80e^{0.2t}$ $t = 10 \rightarrow \frac{dB}{dt} = 80e^2 = 591$ (/day)	B1 B1	awrt
	(iv)	$10000 = 500 + 400e^{0.2t} \rightarrow e^{0.2t} = (23.75)$ $0.2t = \ln 23.75$ $t = 15.8$ (days)	M1 DM1 A1	$e^{0.2t} = k$ take logs: $0.2t = \ln k$ awrt

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<p>6 (i)</p> <p>$(x+2)^2 + x^2 = 10$ $x^2 + 2x - 3 = 0 \rightarrow (x+3)(x-1) = 0$ Points (1, 3), (-3, -1) isw</p> <p>or elimination of x leads to $y^2 - 2y - 3 = 0$, then as above</p> <p>(ii)</p> <p>$m^2x^2 + 10mx + 25 + x^2 = 10$ $(m^2 + 1)x^2 + 10mx + 15 = 0$ $b^2 - 4ac = (0)^2 \rightarrow 100m^2 - 60(m^2 + 1) = 0$ $m = \pm\sqrt{\frac{3}{2}}$ oe isw</p> <p>Alternative solution: $\frac{dy}{dx} = \frac{-x}{\sqrt{10-x^2}}$ or $\frac{dy}{dx} = -\frac{x}{y}$ Result: $y^2 = x^2 + 5y$ after inserted in $y = mx + 5$ Attempt to solve with $x^2 + y^2 = 10$ $y = 2, x = \pm\sqrt{6}$ $m = \pm\frac{3}{\sqrt{6}}$ oe</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>3 term quadratic with attempt to solve both x or a pair both y or second pair</p> <p>attempt to use discriminant on three term quadratic. Allow unsimplified</p> <p>cao \pm is required</p> <p>allow unsimplified</p> <p>Eliminate x or y both</p>
<p>7 (i)</p> <p>$v = 2\cos t + 1$</p> <p>(ii)</p> <p>$2\cos t + 1 = 0$</p> <p>$t = \frac{2\pi}{3}$ or 2.09</p> <p>(iii)</p> <p>$t = \frac{2\pi}{3} \rightarrow x = 2\sin\left(\frac{2\pi}{3}\right) + \frac{2\pi}{3} = 3.83\text{ m}$ $a = -2\sin t$ $t = \frac{2\pi}{3} a = -\sqrt{3} = -\frac{1.73}{4}\text{ ms}^{-2}$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1ft</p> <p>DB1ft</p>	<p>mark final answer</p> <p>equate their v to zero (must be a differential) and attempt to solve to find an angle awrt</p> <p>awrt</p> <p>ft <i>their</i> v (2nd differential)</p> <p>ft using <i>their</i> angle t in correct a awrt</p>
<p>8 (i)</p> <p>$\frac{dy}{dx} = \frac{(2+x^2) \times 2x - x^2 \times 2x}{(2+x^2)^2} = \frac{4x}{(2+x^2)^2}$ $k = 4$</p> <p>(ii)</p> <p>$\int \frac{x}{(2+x^2)^2} dx = \frac{1}{4} \times \frac{x^2}{2+x^2} + (c)$ isw</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p>	<p>apply quotient or product rule unsimplified</p> <p>$k=4$ does not need to be specifically identified</p> <p>$\frac{1}{\text{their } k} \times$ original function</p>

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9	$(a + 3\sqrt{5})^2 = a^2 + 3\sqrt{5}a + 3\sqrt{5}a + 45 \text{ oe}$ <p>Equate: $a^2 + a + 45 = 51$ and $6a - b = 0$</p> $(a + 3)(a - 2) = 0$ <p>$a = -3, 2$ $b = -18, 12$</p>	<p>B1</p> <p>B1 B1</p> <p>M1</p> <p>A1 A1</p>	<p>anywhere</p> <p>Attempt to solve three term quadratic with integer coefficients obtained by equating coeffs</p> <p>Both <i>as</i> correct or one correct pair</p> <p>Both <i>bs</i> correct</p>
10 (i)	$\sec x \operatorname{cosec} x = \frac{1}{\cos x \sin x}$ $\cot x = \frac{\cos x}{\sin x}$ <p>LHS = $\frac{1 - \cos^2 x}{\cos x \sin x}$ oe</p> $= \frac{\sin^2 x}{\cos x \sin x} = \tan x \quad \text{AG}$	<p>B1</p> <p>B1</p> <p>B1ft</p> <p>B1</p>	<p>anywhere</p> <p>anywhere</p> <p>correct addition of <i>their</i> terms</p> <p>use of identity and cancel</p>
(ii)	$3 \cot x - \cot x = \tan x \rightarrow 2 \cot x = \tan x$ <p>$\tan^2 x = 2$ oe $x = 54.7, 125.3, 234.7, 305.3$</p>	<p>M1</p> <p>A1 A1 A1</p>	<p>equate and collect like terms, allow sign errors</p> <p>2 values</p> <p>only 2 more values. awrt</p>
11 (i)	<p>Area of sector = $\frac{1}{2} \times x^2 \times 0.8 (= 0.4x^2 \text{ cm}^2)$</p> <p>$SR = 5 \sin 0.8 (= 3.59)$ or $OR = 5 \cos 0.8 (= 3.48)$</p> <p>Area of triangle = $\frac{1}{2} \times 5 \cos 0.8 \times 5 \sin 0.8 = 6.247 \text{ cm}^2$</p> <p>$0.08x^2 = 6.247$ $x = 8.837 \text{ cm} \quad \text{AG}$</p>	<p>B1</p> <p>B1</p> <p>M1 A1</p> <p>A1</p>	<p>anywhere</p> <p>SR may be seen in stated $\frac{1}{2} ab \sin C$</p> <p>insert correct terms into correct area formulae</p>
(ii)	<p>$SQ = 8.84 - 5 (= 3.84 \text{ cm})$</p> <p>$PR = 8.84 - 5 \cos 0.8 (= 5.35 \text{ or } 5.36 \text{ cm})$</p> <p>$PQ = 8.84 \times 0.8 (= 7.07 \text{ cm})$</p> <p>Perimeter = 19.84 to 19.86 cm or rounded to 19.8 or 19.9</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>two lengths from SQ, PR, PQ awrt</p> <p>third length awrt</p> <p>sum</p>
(iii)	<p>Area $PQSR = 4 \times 6.247$ $= 25 \text{ cm}^2$</p>	<p>M1</p> <p>A1</p>	<p>24.95 to 25</p>

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12 (i)	$f(2) = 3(2^3) - 14(2^2) + 32 = 0$ Or complete long division	B1	
(ii)	$f(x) = (x-2)(3x^2 - 8x - 16)$ $f(x) = (x-2)(x-4)(3x+4)$	M1 A1 M1 A1	$3x^2$ and 16 8x and correct signs Factorise three term quadratic
(iii)	$x = 2, 4$	B1	
(iv)	$\int 3x - 14 + \frac{32}{x^2} dx = 1.5x^2 - 14x - \frac{32}{x} (+ c)$ Area = $\left[1.5x^2 - 14x - \frac{32}{x} \right]_2^4$ = (-) 2	B1 B1 M1 A1	first 2 terms third term correct unsimplified Limits of 2 and 4 and subtract